4.1 Key Terms/Concepts: Define if not listed

- Absolute Maximum/Minimum
- Local Maximum/Minimum
- Critical Number/Point
- The Extreme Value Theorem p. 272 - If f is continuous on a closed interval $[a,b]$, then f has a absolute maximum and minimum.
- Fermat's Theorem p. 273 If f has a local maximum or minimum at a point c and the derivative exists at this point, then $f'(c) = 0$.
- **The Closed Interval Method** p. 275 Fill in the steps:
  1.
  2.
  3.

Exercise 9 p. 277 Sketch the graph of f that is continuous on $[1,5]$ and has an absolute maximum at 5, an absolute minimum at 2, a local maximum at 3, and local minima at 2 and 4.

Exercise 28 p. 277 Sketch the graph of f by hand and use your sketch to find the absolute and local maxima and minima.

$$f(x) = \begin{cases} 4 - x^2 & \text{if } -2 \leq x < 0 \\ 2x - 1 & \text{if } 0 \leq x \leq 2 \end{cases}$$

Exercise 33 p. 277 Find the critical numbers of the following function.

$$f(x) = x^3 + 3x^2 - 24x$$

Exercise 51 p. 179 Find the absolute maximum and minimum values of the function on the given interval.

$$f(x) = x^4 - 2x^2 + 3,\ [-2,3]$$

4.2 Key Terms/Concepts:

- Rolle's Theorem p. 280 - If f is continuous on $[a,b]$, differentiable on (a,b) and $f(a) = f(b)$, then there is a number c in (a,b) such that $f'(c) = 0$.

- The Mean Value Theorem p. 282 - If f is continuous on $[a,b]$ and differentiable on (a,b), then there is a number c in (a,b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a} \iff f(b) - f(a) = f'(c)(b - a)$$

- Thoerem p. 284 - If $f'(x) = 0$ for all x in an interval (a,b), then f is constant on (a,b)

Exercise 4 p. 285 Verify that the function satisfies the three hypotheses of Rolle's Theorem on the given interval. Find all numbers c that satisfy the conclusion of Rolle's Theorem.

$$f(x) = \cos 2x,\ [\pi/8, 7\pi/8]$$

Exercise 16 p. 285 Let $f(x) = 2 - |2x - 1|$. Show that there is no value of c such that $f(3) - f(0) = f'(c)(3 - 0)$. Why does this not contradict the Mean Value Theorem?

Exercise 18 p. 285 Show that the equation $2x - 1 - \sin x = 0$ has exactly one real root.

Exercise 25 p. 286 Does there exist a function f such that $f(0) = -1$, $f(2) = 4$, and $f'(x) \leq 2$ for all x?

4.4 Key Terms/Concepts:

L'Hôpital's Rule:

If $\lim_{x\to c} f(x) = 0 = \lim_{x\to c} g(x)$ OR

$\lim_{x\to c} f(x) = \pm\infty = \lim_{x\to c} g(x)$ AND

$\lim_{x\to c} \frac{f'(x)}{g'(x)}$ exists, then $\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$

Evaluate the following limits:

Exercise 1

$$\lim_{x\to 0} \frac{e^{5x}-1}{\sin 3x}$$

Exercise 2

$$\lim_{x\to 2} \frac{x^3-3x^2-9x+22}{x^3-8x^2+21x-18}$$

Exercise 3 (Stewart #57 p. 305)

$$\lim_{x\to +\infty} \left(1+\frac{3}{x}+\frac{5}{x^2}\right)^x$$

Exercise 4

$$\lim_{x\to 2} \frac{x^2+3x-5-5\tan\left(\frac{\pi x}{8}\right)}{\sin(x-2)\cos(x-2)}$$

4.4 Comprehension Check

*How do we know a function has a limit of infinity at a point?

*What does it mean for a limit to exist?

*Determinate/Indeterminate Form?? What are they?

4.3/4.5 Key Terms/Concepts:

Increasing/decreasing
First Derivative Test
Concave Up/Concave Down
Second Derivative Test

Steps to Curve Sketching:

1. Determine the Domain
2. Locate ALL Intercepts
3. Determine Symmetry (if any)
4. Find any Asymptotes
5. Find the intervals of increase/decrease
6. Determine Local Max/Min Values
7. Find Points of Inflection and intervals of Concavity
8. Sketch the curve

Exercise #12 p 295

Find (a) the intervals of increase/decrease for f, (b) local max/min values of f, (c) intervals of concavity and inflection points.

$$f(x) = \frac{x^2}{x^2+3}$$

Exercise #28 p. 296

Sketch the function such that $f'(x) > 0$ if $|x| < 2$, $f'(x) < 0$ if $|x| > 2$, $f'(2) = 0$, $\lim_{x\to\infty} f(x) = 1$, $f(-x) = -f(x)$, $f''(x) < 0$ if $0 < x < 3$, $f''(x) > 0$ if $x > 3, x < 0$

Exercise #18 p. 315

Use the above steps to sketch the graph of $y = \dfrac{x}{x^3 - 1}$

4.7 Key Terms/Concepts:

Optimization Problem Solving Steps

1. Understand the Problem
2. Draw a Diagram
3. Introduce Notation
4. Express the needed quantity in terms of something else from Step 3
5. Find relationships among other variables
6. Find the absolute min or max.

Exercise #6 p. 328

Find the dimensions of a rectangle whose area is $1000\,m^2$ *and whose perimeter is small as possible.*

Exercise #12 p. 328

A box with a square base and open top must have a volume of $32,000\ cm^3$. *Find the dimensions of the box that minimize the amount of material used.*

Exercise #18 p. 328

Find the point on the line $6x + y = 9$ *that is closest to the point* $(-3,1)$.

Exercise #36 p. 329

A fence 8ft. tall runs parallel to a tall building at a distance of 4ft. from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building?

4.9 Key Terms/Concepts:
Antiderivative
General form of an Antiderivative

Exercise #4 p 345
Find the general antiderivative of $f(x) = 8x^9 - 3x^6 + 12x^2$

Exercise #12 p. 345
Find the general antiderivative of $f(x) = \dfrac{5 - 4x^3 + 2x^6}{x^6}$

Exercise #30 p. 345
Find f if $f'(x) = 8x^3 + 12x + 3, \quad f(1) = 6$

Exercise #44 p. 345
Find f if $f''(x) = 2e^t + 3\sin t, \quad f'(0) = 0, \quad f(\pi) = 0$

Exercise #74 p. 346
A car braked with a constant deceleration of $16\, ft/s^2$*, producing skid marks measuring 200 ft before coming to a stop. How fast was the car traveling when the brakes were first applied?*